

4.7

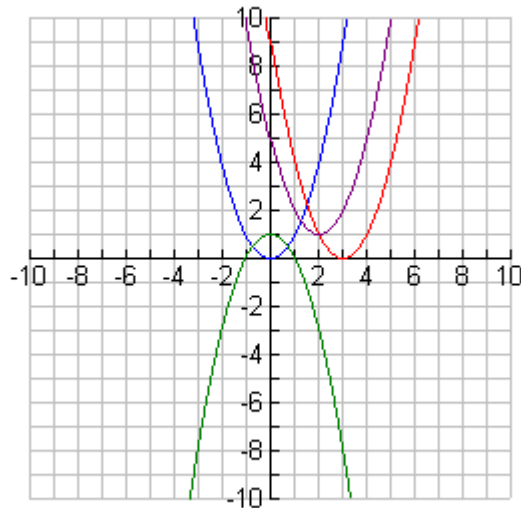
Graphical Transformations

$$y = x^2$$

$$y = (x-3)^2$$

$$y = 1 - x^2$$

$$y = x^2 - 4x + 5$$



Transformations – functions that map real numbers to real numbers

Rigid Transformations – leave the size and shape of a graph unchanged (horizontal and vertical translations, reflections)

Non-rigid transformations – distort shape of graph (horizontal and vertical stretches and shrinks)

Vertical Translations:

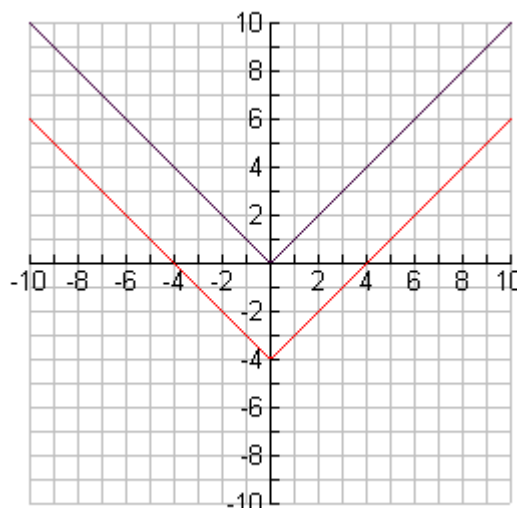
Vertical translation shifts graph up or down.

$y = f(x) + c$ up by c units.

$y = f(x) - c$ down by c units

Ex. $y = |x|$

$$y = |x| - 4$$



Horizontal Translations:

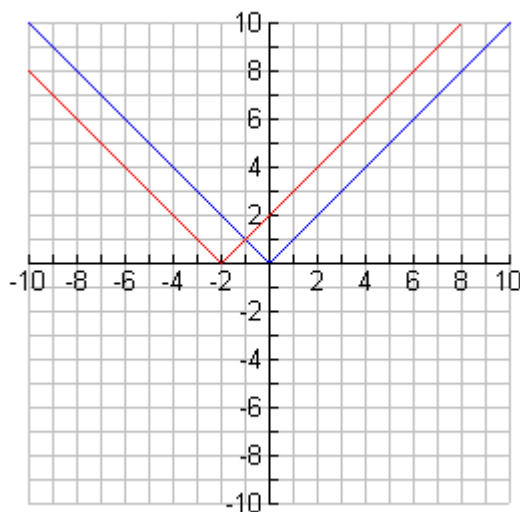
Horizontal translation shifts graph left or right.

$y = f(x - c)$ to right by c units.

$y = f(x + c)$ to left by c units.

$$y = |x|$$

$$y = |x + 2|$$



Reflections:

Points (x, y) and $(x, -y)$ are **reflections of each other across the x-axis**.

Across the x-axis: $y = f(x) \rightarrow y = -f(x)$

Points (x, y) and $(-x, y)$ are **reflections of each other across the y-axis**.

Across the y-axis: $y = f(x) \rightarrow y = f(-x)$

Finding Equations of Reflections:

Find an equation for the reflection of $f(x) = \frac{3x-7}{x^2+2}$ across each axis.

Reflection across x-axis: $y = -f(x)$

$$-f(x) = -\left(\frac{3x-7}{x^2+2}\right) = \frac{-3x+7}{x^2+2}$$

Reflections across y – axis: $y = f(-x)$

$$f(-x) = -\left(\frac{3(-x)-7}{(-x)^2+2}\right) = \frac{-3x-7}{x^2+2}$$

Stretches and Shrinks

Let c be a positive real number. Then the following transformations result in stretches or shrinks of the graph of $y = f(x)$.

Horizontal Stretches or Shrinks

$y = f\left(\frac{x}{c}\right)$ a stretch by a factor of c if $c > 1$

a shrink by a factor of c if $0 < c < 1$

Vertical Stretches or Shrinks

$y = c \bullet f(x)$ a stretch by a factor of c if $c > 1$

a shrink by a factor of c if $0 < c < 1$

Examples:

Transform the given function by a) a **vertical stretch** by a factor of 3, and b) a **horizontal shrink** by a factor of $1/2$.

$$f(x) = |x+4|$$

$$\text{a) } 3f(x) = 3|x+4|$$

$$\text{b) } f\left(\frac{x}{1/2}\right) = f(2x) = |2x+4|$$

Describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

$$y = -2\sqrt{x+3}$$

Starting with $y = \sqrt{x}$, translate left 3 units, vertical stretch by factor of 2, and reflect across x-axis.

A new graph is obtained from the series of transformations on the given graph, write the equation for the new graph.

$y = x^2$; a vertical stretch by a factor of 4, then a shift right 6 units.

$$y = 4(x-6)^2$$

Even function – for all x in the domain of f $f(-x) = f(x)$, symmetric about the y-axis.

$$(x, y) \rightarrow (-x, y)$$

Odd function – for all x in the domain of f $f(-x) = -f(x)$, symmetric about the origin.

$$(x, y) \rightarrow (-x, -y)$$

Examples:

Determine whether the given function is even, odd or neither.

a) $f(x) = x^2 + 4$

Step 1: Find $f(-x)$ and if $f(-x) = f(x)$, then it is even.

$$f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x), \text{ so the function is even. We are done.}$$

Step 2: (If necessary)

If not even then check to see if it is odd, $f(-x) = -f(x)$.

Try $f(x) = x^2 - 2x - 4$

Step 1:

$$f(-x) = (-x)^2 - 2(-)x - 4 = x^2 + 2x - 4 \neq f(x), \text{ not even.}$$

Step 2:

We know from step 1 that $f(-x) = x^2 + 2x - 4$ so now we find $-f(x)$.

$$-f(x) = -(x^2 - 2x - 4) = -x^2 + 2x + 4 \neq f(-x), \text{ so the function is neither even or odd.}$$

Shifty WS.